



## Evenness, Evenness, Monotonicity of the Function

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**Abstract.** When the authors chose the topic "Functions and graphs" and set the goal of creating a collection of data on this topic, they paid special attention to the following: it is known that there are various ways of giving a function. They are analytical method, tabular method, verbal method and graphical method.

**Key words:** function, theorem, inequality, equation, natural numbers, odd function, even function, etc.

Sometimes a graphical method may be the only way to assign a function. It is used in physics, technology, and medicine, and is the basis for the work of many self-written instruments. However, despite the fact that the graphic method of function assignment is widespread, the lack of literature containing a set of information dedicated to the application of mathematical analysis concepts in the verification of functions is clearly visible. If the equality  $f(-x) = f(x)$  is fulfilled in  $\forall x \in B(f)$  for the function  $y = f(x)$  in the set whose detection area is symmetric with respect to the point  $O(0; 0)$ , then  $f(x)$  a function is called an even function, and when the equality  $f(-x) = -f(x)$  is fulfilled, it is called an odd function. For example,  $f(x) = 2x^2 + 3$  is an even function because  $f(-x) = 2(-x)^2 + 3 = 2x^2 + 3 = f(x)$ . Also,  $y = |x|$ ,  $y = x^4$  are also even functions.  $(-x)^5 = -x^5$ , therefore,  $y = x^5$  is an odd function. In general,  $x^{2n}$ ,  $n \in \mathbb{N}$ , functions are even,  $x^{2n-1}$ ,  $n \in \mathbb{N}$ , functions are odd functions. According to the definitions, the graph of the odd function is symmetrical with respect to the coordinate origin, and the graph of the even function is symmetrical with respect to the ordinate axis. Even and odd functions are defined symmetrically with respect to the coordinate origin. In checking the functions for evenness, it is as follows: a) let the function  $f(x)$  be defined in  $D(f)$ , and the function  $g(x)$  be defined in  $D(g)$ . If the functions  $f(x)$  and  $g(x)$  are even (or odd) in the general definition domain  $x \in D(f) \cap D(g)$ , then their sum  $(f + g)(x)$  is also even will be (odd). Indeed,  $(f + g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f + g)(x)$ ;  $(f + g)(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f + g)(x)$ ; b) the product of two even (odd) functions is an even function, and the product of odd and even functions is an odd function. Indeed, if  $f$  and  $g$  are even functions, then  $(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x)$ . The rest of the cases are proved in the same way.

The evenness and exactness of a function is one of its main properties, and evenness occupies an impressive part of the school course in mathematics. This largely clarifies the nature of the function's behavior and greatly simplifies the construction of the corresponding graph. Let's define the parity of the function. In general, the studied function is considered even if the corresponding values of  $y$  (function) are equal for the opposite values of the independent variable ( $x$ ) located in its domain of definition. From the above definition follows a necessary condition for the domain of such a function, i.e., symmetry with respect to the point  $O$ , which is the origin of the coordinates, because if any point  $b$  exists in the domain of definition, even if it is a function, the corresponding point  $-b$  also lies in this domain. By the way, it should be remembered that there are functions that cannot be classified according to these criteria, they are called neither even nor odd. Function parity can be used to solve equations.

To solve an equation such as  $g(x) = 0$ , where the left side of the equation is an even function, it is enough to find its solutions for non-negative values of the variable. The obtained roots of the equation must be combined with opposite numbers. One of them should be checked. The same is successfully used to solve non-standard problems with the parameter. For example, is there a value for the parameter  $a$  that makes the equation  $2x^6 - x^4 - ax^2 = 1$  three roots? If we consider that the variable enters the equation in even degrees, it is obvious to replace  $x$  by  $-x$ . The given equation does not change. It follows that if a given number is its root, then so is its opposite number. The conclusion is obvious: the non-zero roots of the equation are included in the set of its solutions in the form of "pairs". It is clear that the number  $0$  is not itself, that is, the number of roots of such an equation can be only even, and naturally, for any value of the parameter, it cannot have three roots. But the number of roots of the equation  $2^x + 2^{-x} = ax^4 + 2x^2 + 2$  can be odd and for any value of the parameter. In fact, it is easy to check the set of roots for a given equation containing "even" solutions. Let's check if  $0$  is a root. Substituting it into the equation, we get  $2 = 2$ . Thus, in addition to "even",  $0$  is also a root, confirming their odd number.

The mapping of an element  $x$  from set  $A$  to an element  $y$  from set  $B$  is called a reflection. If each element of the set  $A$  corresponds to each element of the set  $B$ , then the set  $A$  is called mirrored to the set  $B$ . Definition 2. The mapping of each  $x$  element in set  $A$  to a specific  $y$  element in set  $B$  based on some law or rule is called a function and is defined as  $y = f(x)$ . Here,  $x$  is an arbitrary variable or argument, and  $y$  is an arbitrary variable or function. For example, the dependence of the path on the speed, or the dependence of the speed on the acceleration Example 1. If the car travels  $60$  km per hour, write the equation of its path. Solution: if he walks  $60$  km in one hour, then he walks  $120$  km in two hours, so  $s = 60$ . Since the ordinate of point  $A$  is smaller than the ordinates of other points, this point is called the smallest value of the function in the given section. In our example, the smallest value of the function is  $-2$ . Similarly, the ordinate of the point  $D$  is greater than the ordinate of the other points, so the maximum value of the function is  $2$ . Now pay attention to the point  $B$ , the ordinate of the point  $B$  is greater than all the points around it, so the point  $B$  is called the maximum point of the function. Similarly, the ordinate of point  $C$  is smaller than all points around it, so point  $B$  is called the minimum point of the function. Periodic function. In nature and in practice, there are certain processes that repeat over time. For example, every  $T = 12$  hours, the clock shaft rotates once, and the position where it was at a moment of time  $t$  before!  $t + T$ ,  $t + 2T$ , in general, returns to this place at time instants. The distance between the Sun and the Earth changes during  $T = 1$  year, and the change is repeated in the second year. In general, there exists a number  $T$  such that  $y = f(x)$  for any  $x$  taken from the domain of definition  $D(f)$  of the function  $x + T$ , if the numbers  $x - T$  also belong to  $D(f)$  and  $f(x) = f(x + T) = f(x - T)$  if the equations are fulfilled, the function  $f$  is a continuous function, the number  $T$  is the period of this function, and the smallest positive period is called the main period of the function.

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